Probing 1D super-strongly correlated dipolar quantum gases

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One-dimensional (1D) dipolar quantum gases are characterized by a very special condition where super-strong correlations occur to significantly affect the static and dynamical low-energy behavior. This behavior is accurately described by the Luttinger Liquid theory with parameter $K < 1$. Dipolar Bose gases are routinely studied in laboratory with Chromium atoms. On the other hand, 1D realizations with molecular quantum gases can be at reach of current experimental expertises, allowing to explore such extreme quantum degenerate conditions which are the bottom line for designing technological devices.

Aim of the present contribution is to focus on the possible probes expected to signal the reach of Luttinger-Liquid behavior in 1D dipolar gases.

I. INTRODUCTION

One-dimensional (1D) dipolar quantum gases have been shown to always be in a very special condition where super-strong correlations occur to significantly affect the system behavior [1]. In this strongly-correlated regime, 1D dipolar gases are described by the Luttinger-Liquid theory with Luttinger parameter $K < 1$, as we have demonstrated by comparing the results of Luttinger Liquid theory with Quantum Monte Carlo simulational data for static correlation functions. In fact, the dipolar tail of the interactions greatly enhances the correlation effects, which are also observable at the dynamical level [2].

Dipolar Bose gases are routinely realized in laboratory with Chromium atoms [4, 5] and more recently the possibility of reducing the effects of short range interactions with Fano-Feshbach resonances has been reported, so that a purely dipolar character emerges [6]. Moreover, 1D dipolar quantum gases may soon be within reach of current experimental expertises with molecular trapping. Orders of magnitudes provided in Ref. [1] suggest that molecular dipolar gases such as SrO [7] are promising candidates to explore the whole crossover from the Tonks-Girardeau regime with $K = 1$ down to the quasi-ordered limit with $K \ll 1$.

The interest in realizing such extreme 1D quantum-degeneracy conditions accompanied by strong correlations is twofold. Such conditions would provide a real laboratory to test early theoretical predictions, and are the bottom line for designing technological devices for quantum information processing [8], precision measurements [9], and atomtronic devices [10, 11].

While for the detailed theoretical and numerical characterization of the physics of 1D dipolar gases we refer to our more recent review [3], the aim of the present contribution is to focus on the possible probes which are expected to signal the reach of Luttinger-Liquid behavior in 1D dipolar gases. After outlining the system under consideration in Sec. II, we then proceed to discuss in Secs. III,IV,V three different types of probes, two of them being already illustrated in detail in [1, 2, 12].

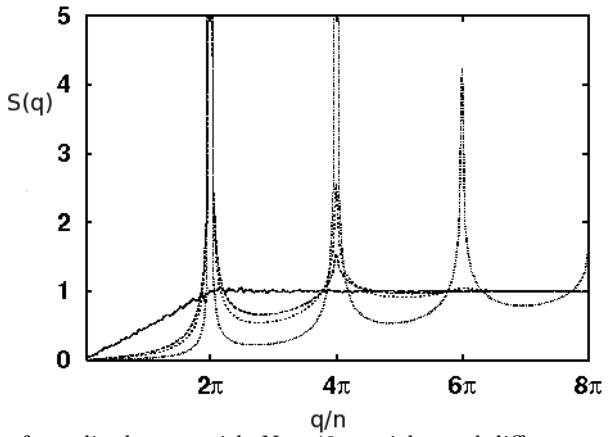

FIG. 1: $S(q)$ in dimensionless units for a dipolar gas with $N = 40$ particles and different values of $nr_0 = 0.01, 50, 100$ and 1000. Decreasing slopes as $q \to 0$ and the emergence of additional peaks correspond to increasing nr_0 values.

II. THE SYSTEM

The Hamiltonian modelling the system under consideration is that of N atoms located at position x_i with mass M and permanent dipoles moments along and orthogonal to a line, yielding purely repulsive interactions:

$$H = -\frac{1}{r_s^2}\sum_i \frac{\partial^2}{\partial x_i^2} + \frac{1}{r_s^3}\sum_{i<j}\frac{1}{|x_i - x_j|^3} \tag{1}$$

in effective Rydberg units $Ry^* = \hbar^2/(2Mr_0^2)$. The effective Bohr radius $r_0 \equiv MC_{dd}/(2\pi\hbar^2)$ is build up from the interaction strength $C_{dd} = \mu_0\mu_d^2$ for magnetic and $C_{dd} = d^2/\epsilon_0$ for electric dipoles. The interacting regime is governed by the usual parameter $r_s = 1/(nr_0)$ with n the linear density, so that large densities correspond to strong correlations.

We study the ground-state properties as well as the static and dynamic low-energy behavior of this system by combining theoretical and numerical methods. These are in turn based on bosonization and Reptation Quamtum Monte Carlo (RQMC) techniques with finite number of particles, respectively. Details of the calculations can be found in Refs. [1, 2].

III. STATIC PROBES

One possible outcome of the RQMC numerical simulations is the static structure factor $S(k)$, which in terms of the density operator $\hat{n}$ is $S(q) = \int_0^L dx e^{-iqx}\langle\hat{n}(x)\hat{n}(0)\rangle$.

Fig. 1) [1] displays the overall structure at different densities values. Free fermion-like behavior as in the Tonks-Girardeau gas sets in at low density ($nr_0 = 0.01$ in the graph). Quasi-Bragg peaks appear at reciprocal lattice vectors $q/n = 2\pi m$ with m integer. Their number increases with increasing the density nr_0, until a quasi-ordered state sets in. Scaling studies of the RQMC data at different values of N allow to conclude that the system is a Luttinger liquid.

In particular, a power-law decay of the peak heights in $S(q)$ and a divergence of their intensity with the system size is observed. In the Luttinger Liquid theory, this reflects into a flattening of the momentum distribution $n(q)$ as $nr_0 \gg 1$. Both of these fingerprints of the Luttinger-liquid behavior can be probed in experiments with atomic gases. In particular, the static structure factor can be accessed by means of Bragg scattering techniques [13], while the momentum distribution $n(q)$ can be inferred from analysis of time-of-flight density images.

But the easiest way to probe the Luttinger liquid behavior in the whole crossover is to measure the eigenfrequencies of the collective trap modes, after modulating the trap by an external field with appropriate symmetry and as done in the very early seminal experiments with atomic gases [14, 15]. In Ref. [12] we have shown that for a system trapped in a shallow potential, an effective Luttinger-liquid hydrodynamic theory can be set up, discussing in detail the general equations and their range of validity and the results following the use of Local Density Approximation. Within this approximation we have determined the lowest collective modes of the harmonically trapped dipolar gas in the crossover. Figure 2 displays for example the frequencies of the lowest, breathing mode, which can be easily excited by periodically compressing the trap. This prediction enables *e.g.* the identification of the interaction regime of the trapped dipolar gas and provides a further testing of our theory.

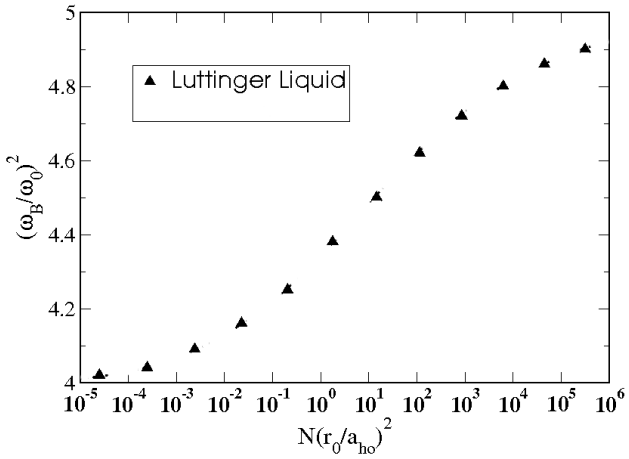

FIG. 2: Squared frequency ω_B^2 of the breathing mode scaled to the trap frequency ω_0 *vs.* the interaction parameter $N(r_0/a_{\rm ho})^2$, as calculated from the Luttinger-liquid hydrodynamics (LL) [12].

IV. DYNAMICAL PROBES

The energy of the low-lying excitations are extracted from the RQMC data for the imaginary-time density-density correlation function $F(q,\tau)$, after exploiting the Laplace transformation connecting $F(q,\tau)$ to the dynamical structure factor, that is $F(q,\tau) = \int_0^\infty d\omega \exp(-\omega\tau) S(q,\omega)$. After checking that different specific choices of $S(q,\omega)$ at zero temperature, corresponding to different approximations, do not lead to significant changes in the quantitative results, we assume the form

$$S(q,\omega) = \sum_i^\alpha A_i(q)\delta(\omega - \omega_i) \tag{2}$$

and within this choice $F(q,\tau) = \sum_{i=1,\alpha} A_i(q) e^{\omega_i(q)\tau}$, where ω_i is the excitation modes energy and α is the number of modes needed to yield the best χ^2 in the fitting procedure, and depends on the values of q and nr_0.

Fig. 3 displays the lowest excitation energies $\omega(q)$, while an accurate size effect analysis of the gap sizes as shown in Fig. 4 for $\omega(q/n = 2\pi)$ at different densities $nr_0 = 0.01, 0.1, 1, 10$ and 1000, demonstrate a linear scaling $\omega_N(q = 2\pi) = c(nr_0)/N$ with the slope $c(nr_0)$ an increasing function of nr_0. The overall result is that the gap is closed at all densities. This means that no roton gap appears at the first star of the reciprocal lattice $q = 2\pi$, implying the absence of superfluidity in this system, according to the Landau criterion. Once more, future experiments by means of *e.g.* Bragg spectroscopy techniques [13] may reveal this behavior, thereby providing a test for the present theory.

V. INDUCED OPACITY BY A WEAK BARRIER

We now turn to the third and last possible method to probe the occurrence of strongly repulsive Luttinger Liquid behavior in 1D dipolar gases, which exploits an earlier idea from Refs. [16] [17] [18] [19].

Let us consider a 1D Luttinger Liquid with parameters u and K and a weak local potential barrier with height V_0. The Hamiltonian of the system reads:

$$H = \int \frac{dx}{2\pi}\left[uK(\pi\Pi)^2 + \frac{u}{K}(\partial_x\phi)^2\right] + \frac{V_0}{\pi\alpha}\int dx \sum_{m>0} A_m \cos 2m\phi(0) \tag{3}$$

,

where $[\phi(x), \Pi(x')] = i\delta(x-x')$, α is a short distance cutoff, and A_m are non-universal amplitudes. We define $V_m = V_0 A_m$. The last term in Eq.(3) represents a backscattering operator in the usual fermionic language. A renormalization group calculation[16] shows that:

$$\frac{dV_m}{d\ell} = (1 - m^2 K)V_m, \tag{4}$$

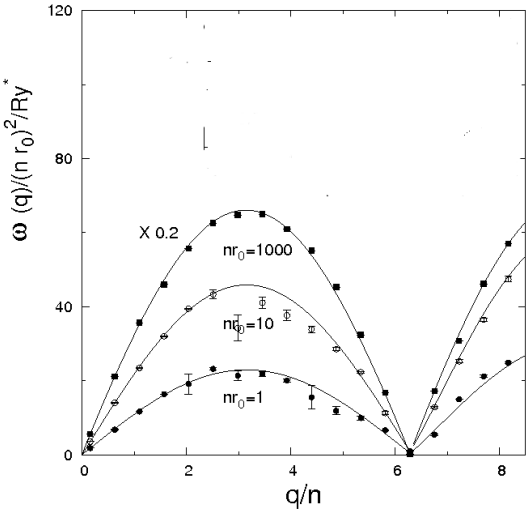

FIG. 3: Lowest excitation energies $\omega(q)$ in Ry^* units and scaled by $(nr_0)^2$, for a dipolar gas with $N = 40$ and different values of $nr_0 = 1, 10$ and 1000 as in the legend. Symbols with error bars are energies extracted using (2), the solid line is a guide to the eye. The curve at $nr_0 = 1000$ is depressed by a factor of 5 for graphical reasons.

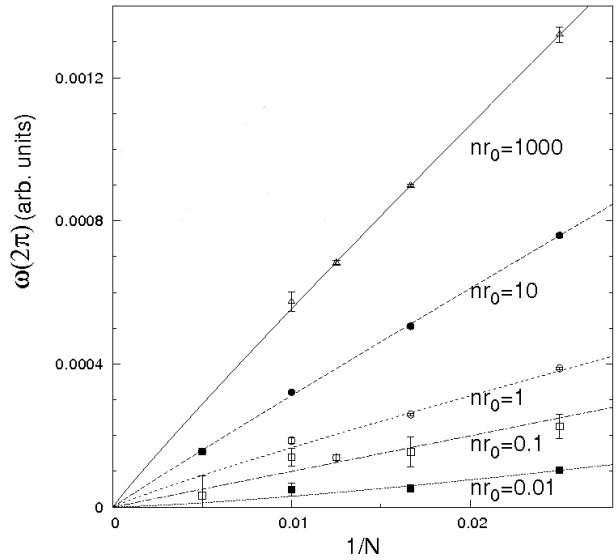

FIG. 4: $1/N$ scaling of $\omega(q = 2\pi)$ in arbitrary units at $nr_0 = 0.01,\ 0.1,\ 1,\ 10$ and 1000 as in the legend. Symbols with error bars: RQMC data. Solid lines: fit to the data.

where $\alpha(\ell) = \alpha e^{\ell}$ is the running cutoff and ℓ the length scale. For $K > 1$, all the V_m decay exponentially to zero, indicating that the barrier is becoming transparent. For $K < 1$, V_1 is growing to infinity. This indicates that there is a strong coupling fixed point, where the phase field $\phi(0)$ is locked to the value of $\pi/2$ in order to minimize the potential energy[16]. The boundary condition $\phi(0) = \pi/2$ means that the system becomes equivalent to two independent semi-infinite Luttinger liquids[20], with total reflection of the particles on the boundary. The barrier is thus completely opaque for $K < 1$. The case $K = 1$ is marginal. Since for $K = 1$ the system is equivalent to free fermions, it is easy to show that in that case there is a reflection and a transmission coefficient across the barrier.

One can thus imagine to push slowly a dipolar atomic beam along a 1D waveguide, against a barrier of appropriate height V_0 and simply observe whether the barrier is opaque or not to the passage of the beam. Since the dipolar gas always have $K < 1$, the barrier should in fact always be opaque, in contrast with the case of a Lieb-Liniger gas, with pure contact interaction, for which $K > 1$ and the barrier is transparent. The opacity of the weak barrier could be

probed in experiments by standard absorption or yet in situ phase contrast imaging of the condensate[21].

VI. CONCLUSIONS

In conclusion, we have discussed a few experimentally feasible methods which can be used to probe the occurence of super-strongly correlated behavior in 1D dipolar quantum gases. In particular, we have identified the fingerprints of such behavior in possible measurements of static and dynamic structure factors, of the momentum distribution, of the collective trap modes, and of the analog of the induced-transparency mechanism. Ongoing experiments with molecular dipolar gases may reveal one or more of these predictions.

Acknowledgements

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